RESEARCH DEPARTMENT

The vertical radiation patterns of high-frequency curtain arrays on plateau sites

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THE VERTICAL RADIATION PATTERNS OF HIGH-FREQUENCY CURTAIN ARRAYS ON PLATEAU SITES

Report No. E-084

(1963/26)

P. Knight, B.A., A.M.I.E.E.
R.E. Davies, M.A.
R.G. Manton, Ph.D., B.Sc., Grad.I.E.E.

W Proctor Wilson

(W. Proctor Wilson)

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SUMMARY

A method for calculating the vertical radiation pattern of a high-frequency curtain array erected on a flat hill or plateau is described; computations showing how this pattern depends on the steepness of the side of the plateau, and on the distance of the array from its edge, are discussed.

1. INTRODUCTION

The type of high-frequency (h.f.) array considered in this report is that used for virtually all of the BBC overseas services, namely a vertical curtain containing a number of tiers, each comprising collinear half-wave radiators. The vertical spacing between the tiers is $\lambda/2$, where λ is the wavelength, and the tiers carry equal co-phased currents.

The shape of the vertical radiation pattern (v.r.p.) of such an array is determined to an important extent by interference between direct and ground-reflected radiation, and consequently depends on the profile of the site on which the array is erected. Ideally, the ground should be perfectly flat for many wavelengths in the direction of propagation, but in practice this condition cannot always be satisfied. The investigation described in this report was therefore carried out to determine the extent to which the performance of a h.f. array is modified by the presence of sloping ground in its vicinity.

The array was assumed to be erected on level ground at the top of a plateau, as shown in Fig. 1; this case is of special interest because a number of BBC arrays are so situated. The proximity of the edge of the plateau can cause the v.r.p. of the array to be distorted, especially at low angles, since reflexion then takes place mainly at the more distant lower ground. This causes the first lobe of the v.r.p. to be at a lower angle than that intended.

Radiation patterns were calculated for the idealized case shown in Fig. 1, in which the ground contours are represented by three planes, two of which are semi-infinite. The edge of the plateau is normal to the plane of the paper, and assumed to extend to infinity; the line sources forming the array are also assumed to be parallel to this edge. The radiation pattern normal to the array (i.e. in the plane of the paper) is then independent of the width of the array, and the latter may for convenience be assumed to be infinite. This enables the problem to be solved by a two-dimensional method, with a considerable simplification of the mathematics.

The ground was assumed to be perfectly-conducting, i.e. to have a reflexion coefficient of -1; this is a good approximation for horizontally-polarized radiation at high frequencies, and was convenient inasmuch as it greatly simplified the computer programme; there would, however, be no fundamental difficulty in incorporating an angular-dependent reflexion coefficient in the programme.

The theoretical method used for calculating the v.r.p. is described in Section 2; it was programmed for a digital computer, the four parameters a, b, c and h (Fig. 1), and the number of tiers, being considered as variables. As stated above, the vertical spacing between the tiers was assumed to be $\lambda/2$ and the tiers were assumed to carry equal currents, since this arrangement is nearly always used in BBC arrays. Computations for other spacings and for unequal currents in the tiers would, however, be equally possible.

2. THEORETICAL METHOD

Reflexion from the perfectly-conducting ground planes of Fig. 1 is equivalent to radiation from a number of images as shown in Fig. 2. In this diagram the planes are labelled P_1, P_2 and P_3, the position of the source being denoted by S, and the positions of its images by S_i, where S_i is the image of S in P_i. By extension, S_{ij} is the image of S_i in P_j and corresponds to a double reflexion. The images may be regarded as separate sources; throughout this report the term 'source' should be understood as referring either to the actual source or to any of its images.

Although an exact solution for the field distribution of such a two-dimensional arrangement of sources and planes is theoretically possible, the practical difficulties involved are formidable; it was therefore necessary to resort to a number of approximations. The validity of these approximations was subsequently tested by comparing a number of computed patterns with measurements made on a small-scale model.

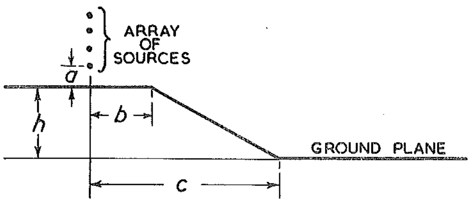

Fig. 1 - An array on a plateau

The usual approximations that sources carrying equal currents contribute equally to the resultant field strength, and that lines drawn from them to the point of observation are all parallel, may be made in this case since we are concerned with the calculation of the distant field strength. The simplest method of computing the effect of the ground profile makes use of optical ray theory, a set of images being formed by reflexion in the ground. However, since each image contributes to the v.r.p. over a limited range of angles only, a pattern calculated on this basis exhibits discontinuities of amplitude which may be quite large.[1] Although it is possible to draw a smooth curve through the discontinuities, this process can give rise to considerable errors.

The discontinuities in the ray-theory v.r.p. are caused by the changes in ground slope at the top and bottom of the plateau. They were eliminated by assuming that diffracting edges exist at these points, each edge being so arranged that it obscures the region which is not directly illuminated by the source being considered. The effects of the diffracting edges were calculated by the classical Fresnel diffraction theory, which itself involves a number of approximations.

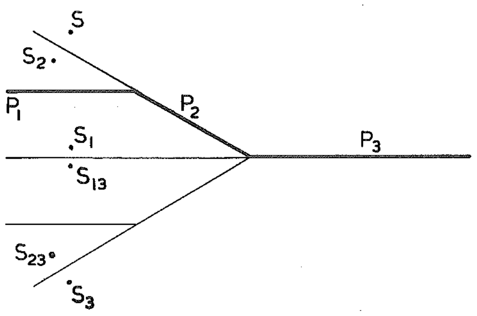

Fig. 2 - Positions of source and images

From Fig. 2 it is seen that, in the ray-theory method, only the sources S, S_1, S_2, S_3 and S_{23} have to be considered, S_2 and S_{23} being omitted if S is below the line of P_2. However, in the diffraction method a number of other sources must also be considered, since radiation is obtained in shadow zones as well as in directly-illuminated zones. Of all the possibilities, only the above five sources and S_{13} are likely to give contributions of reasonable magnitude.* Although S_{13} is always in shadow it must be included to give symmetry about P_3; if it is disregarded considerable errors arise when dimension *h* (Fig. 1) tends to zero.

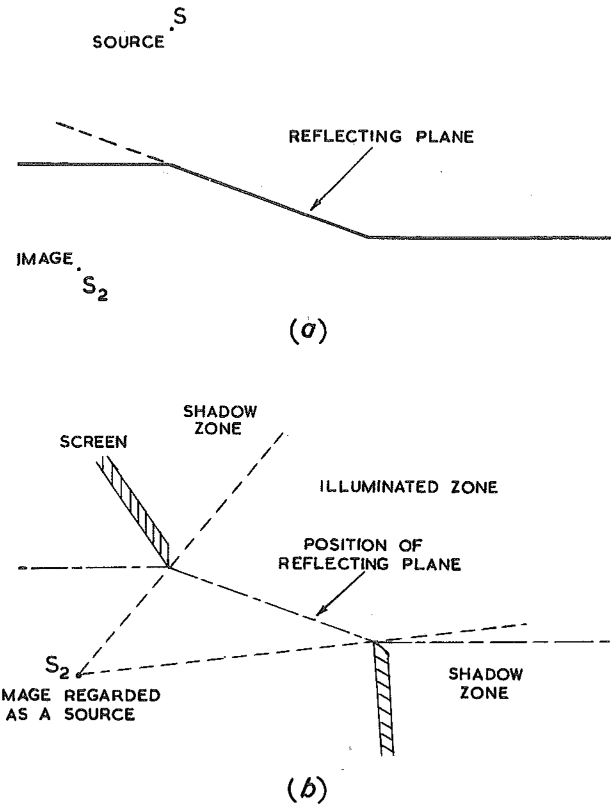

Fig. 3 - Reflexion from a finite plane

(a) actual arrangement

(b) a source seen between a pair of diffracting screens

In order to explain the diffraction method it is convenient to consider the radiation from a single source. Fig. 3 shows how the effect of reflexion from a plane which does not extend to infinity may be regarded as equivalent to direct radiation from a source seen between a pair of diffracting screens, as shown in Fig. 3(b); only one screen is required if the reflecting plane extends to infinity in one direction. A different arrangement of screens is required for each source. Thus in the problem under consideration the six different arrangements shown in Fig. 4 are required; these arrangements differ slightly, depending on whether the source S is above or below the projection of the sloping plane. It will be seen that the effect of a diffracting screen has been taken into account when the direct radiation from

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*S_{12} and S_{123} might have been included, but they are both deep in the shadow region and form a symmetrical pair - they can therefore be omitted.

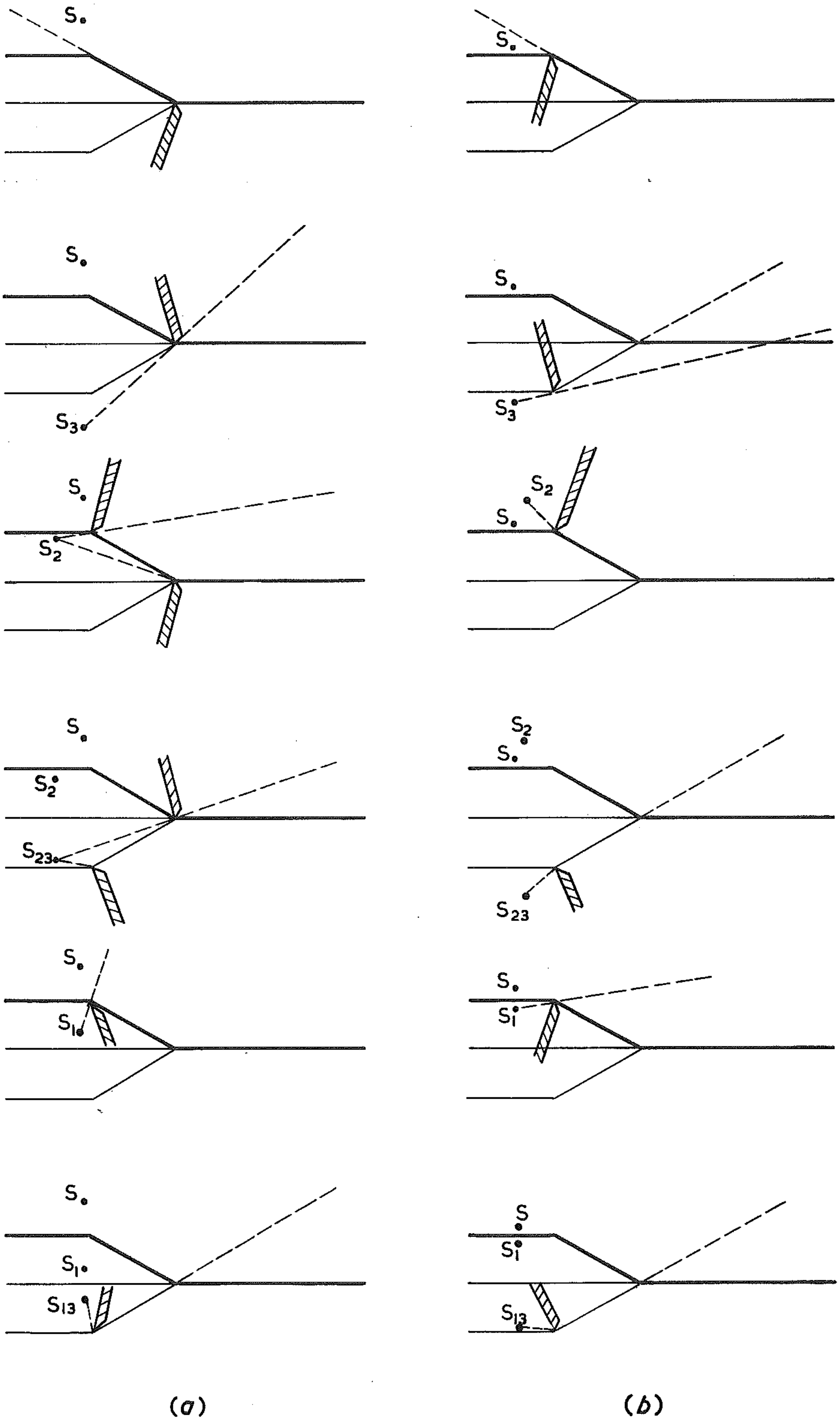

Fig. 4 - Arrangements of sources and diffracting screens

(a) source above projection of sloping plane
(b) source below projection of sloping plane

the source S is being considered, even though the screen does not obstruct the radiation. Its presence is required in order to preserve the symmetry within the complete set of sources and screens; if it were omitted errors would occur at low angles with gently-sloping plateau sides.

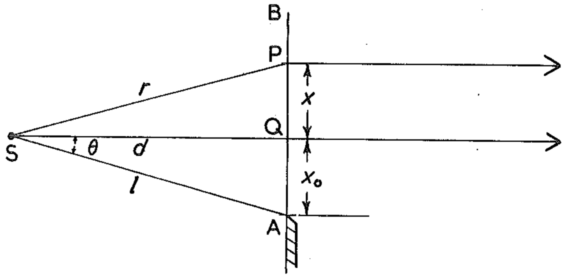

Fig. 5 - A source in the presence of a diffracting screen

Fig. 5 shows a source S in the presence of a diffracting screen. In computing the distant field strength by Fresnel's method, the wavefront AB is regarded as an array of point sources whose phases depend on their distances from S. Since the phase of the contribution from the point P is retarded in phase by $\beta(r - d)$ radians (where $\beta = 2\pi/\lambda$) relative to the contribution from the point Q, the contribution from an element of height δx of the wavefront is:

$$e^{-j\beta(r-d)}\,\delta x$$

The field strength at a distant point due to the whole of the wavefront is therefore proportional to:

$$\int_{-x_o}^{\infty} e^{-j\beta(r-d)}\,dx$$

where x_o is defined in Fig. 5.

Making the usual approximations of Fresnel diffraction theory* we use the substitution:

$$r - d \simeq \frac{x^2}{2d}$$

even though this only applies for small values of x. The integral then becomes:

$$\int_{-x_o}^{\infty} e^{-j\beta x^2/2d}\,dx$$

If we now make the substitution:

$$x = v\left[\frac{\lambda d}{2}\right]^{1/2}$$

the integral may be written in the form:

$$\int_{-v_o}^{\infty} e^{-j\pi v^2/2}\,dv \quad = \quad \tfrac{1}{2} + C(v_o) \;-j\left[\tfrac{1}{2} + S(v_o)\right]$$

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*These approximations may be shown to produce negligible errors provided d is greater than 5λ. In some of the computations described in this report, however, some of the sources are less than 5λ from the nearest diffracting edge. Nevertheless the errors are still small because the range of integration is either restricted to small values of x (Fig. 5) or to a range of values approaching infinity; in the latter case the observer is either well into the illuminated region or well into the shadow region, and the edge of the screen has little effect on the resultant field.

where $$v_0 = x_0 \left[\frac{2}{\lambda d}\right]^{1/2} \simeq \theta \left[\frac{2l}{\lambda}\right]^{1/2}$$

and $C(v)$ and $S(v)$ are the real and imaginary parts of the Fresnel integral; its value may be obtained from tables or calculated from the series expansion.

When two diffracting screens are present, as in Fig. 6, the integral becomes:

$$\int_{-v_1}^{v_2} e^{-j\pi v^2/2}\, dv = C(v_1) + C(v_2) - j\left[S(v_1) + S(v_2)\right]$$

where $$v_1 = \theta_1 \left[\frac{2l_1}{\lambda}\right]^{1/2} ; \quad v_2 = \theta_2 \left[\frac{2l_2}{\lambda}\right]^{1/2}$$

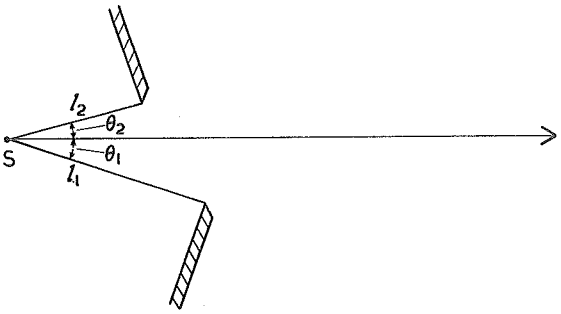

Fig. 6 - A source in the presence of two diffracting screens

It will be seen that the integral representing the distant field strength has been expressed in terms of the two parameters θ and l; these parameters were therefore evaluated as a preliminary step in the computation. Evaluation of the Fresnel integral then gave the magnitude and phase of the field in any particular direction, referred in phase to the position of the source itself. The fields due to all the sources were then referred to a common origin, and added vectorially to give the total field strength due to direct and ground-reflected radiation.

3. EXPERIMENTAL VERIFICATION OF THEORETICAL METHOD

Several checks were made to verify the accuracy of the computation. In one of these, the parameter defining the height of the plateau was read into the computer as zero, and the resulting pattern was found to agree accurately with the v.r.p. for a flat site; this test showed that the effects of all the diffracting edges had been correctly assessed for ground that is almost level. With higher and steeper plateaux, however, there was some possibility that the approximations inherent in Fresnel diffraction theory might cause errors. A number of computed patterns for simple aerials on such sites were therefore verified experimentally using a small-scale model, by the method shown in Fig. 7. The necessity for an extensive ground plane was avoided by modelling both the plateau and its image, the aerial and its image being represented by a pair of dipoles driven in antiphase. The portions of the planes extending behind the dipole were omitted, since currents induced in them contribute insignificantly to the total field for values of Δ (Fig. 7) less than 30°.* For the same reason, in a practical case we need only assess the effect of those ground irregularities which are situated between the source and the service area.

The model was erected so that the edges of the plateau were vertical, pattern measurements being made by rotating the whole arrangement about a vertical

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*This statement may be justified theoretically by calculating the radiation pattern of a dipole near the edge of a half-plane. In directions away from the edge the pattern is almost identical with that of a dipole mounted at the same height above a whole-plane. For examples of such patterns see 'Radio Aerials' by Moullin, Chapter V.

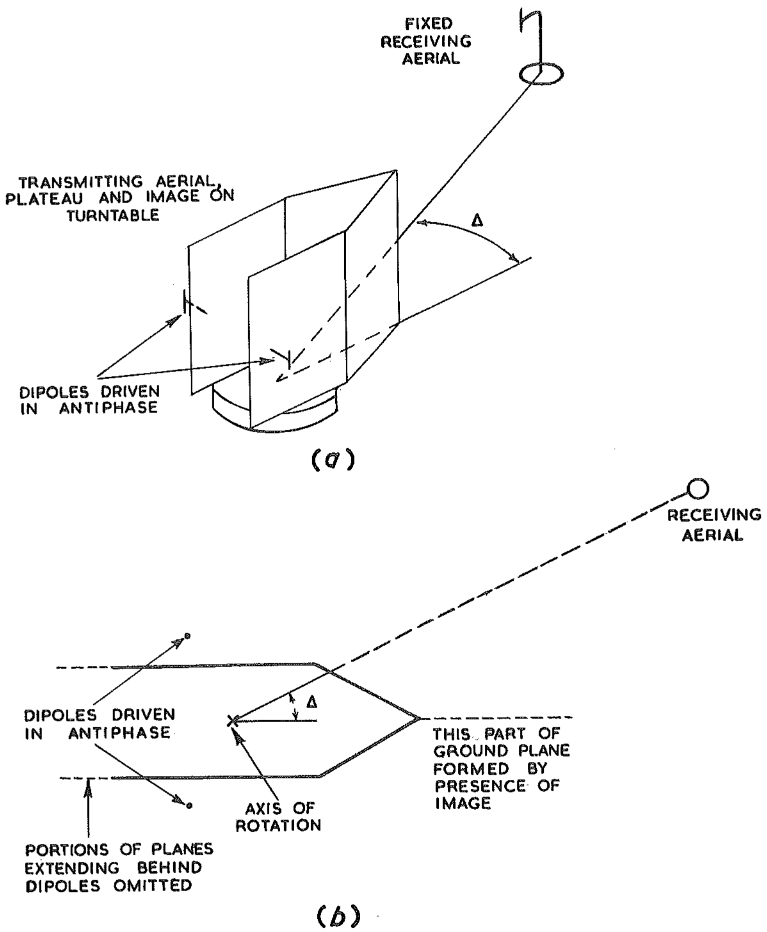
FIXED RECEIVING AERIAL
TRANSMITTING AERIAL, PLATEAU AND IMAGE ON TURNTABLE
Δ
DIPOLES DRIVEN IN ANTIPHASE
(a)
RECEIVING AERIAL
DIPOLES DRIVEN IN ANTIPHASE
Δ
THIS PART OF GROUND PLANE FORMED BY PRESENCE OF IMAGE
AXIS OF ROTATION
PORTIONS OF PLANES EXTENDING BEHIND DIPOLES OMITTED
(b)

Fig. 7 - Experimental arrangement
(a) general arrangement
(b) plan view

axis. The edges of the plateau should ideally have been infinitely long, but this was clearly impossible; they were therefore made sufficiently long for errors due to their finite length to be negligible.

Computed and measured radiation patterns of four different arrangements were compared. In each case the position of the dipole was such that, on a ray-theory basis, the v.r.p. is formed by reflexion in the lower plane at low angles, and by reflexion in the upper plane at high angles; the transition between low-angle and high-angle radiation then occurs at the angle at which the ray from the source strikes the edge of the upper plane. A discontinuity in the ray-theory v.r.p. therefore occurs at this angle. The four arrangements were so chosen that four different types of discontinuities were examined. These were:

(a) Transition from a minimum in the low-angle radiation to a maximum in the high-angle radiation.

(b) Transition between maxima in both low- and high-angle radiation.

(c) Transition from a maximum in the low-angle radiation to a minimum in the high-angle radiation.

(d) Transition between minima in both low- and high-angle radiation.

The ray theory v.r.p. s for these four cases are illustrated in Figs. 8(a) to 8(d) respectively. As the discontinuities in cases (a) and (c) are large, these provide a particularly severe test of the diffraction theory method of computation.

A preliminary experiment was performed to ascertain that the two dipoles carried equal and opposite currents. They were then mounted on the model plateau and radiation patterns measured on both sides of the plane of symmetry. The computed and measured patterns were made to coincide at high angles above the horizontal because, in these directions, there is no uncertainty about the computed v.r.p. This enabled the measured and computed v.r.p. s to be compared directly at lower angles, and not simply scaled for the best fit. The results of this comparison can also be seen in Fig. 8; it will be seen that good agreement was obtained in all four cases.

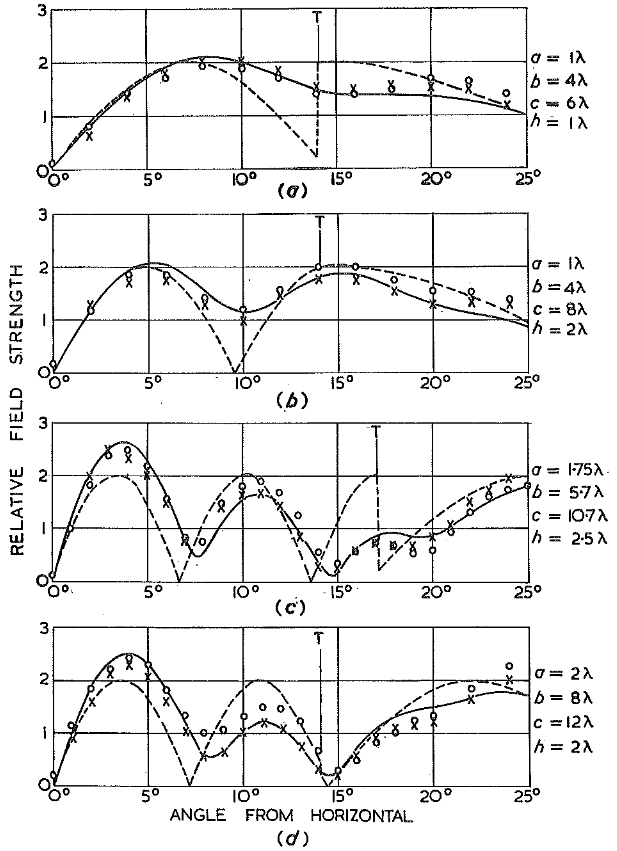

Fig. 8 - Comparison of measured and computed vertical radiation patterns

——— patterns computed by the diffraction method

— — — patterns calculated by ray theory

o X measured points. The two symbols correspond to points measured on the two sides of the plane of symmetry

T angle at which transition between low-angle and high-angle radiation takes place

values of *a*, *b*, *c* and *h*, defining dipole position and plateau shape (see Fig. 1) are stated on right of curves

The model plateau was considerably steeper than those likely to be encountered at h.f. transmitting stations. Since the computed v.r.p. s were found to be reasonably accurate for the steep model plateau and also without error for plateaux of zero height, there was no reason to doubt their accuracy for those of intermediate steepness. The diffraction method was therefore used with confidence to calculate the v.r.p. s of a number of typical arrays on irregular sites.

4. RESULTS OF COMPUTATIONS

All the computations were restricted to plateaux 3·5 wavelengths high. This height was chosen because it corresponds to the difference in height between the aerial site at Daventry and the country to the east of the station. All the patterns were computed for Hm/4/1 arrays* since a large proportion of BBC arrays are of this type.

Fig. 9(a) shows the theoretical v.r.p. of this type of array over flat ground. Now if the height of the array above the ground were increased by 3·5

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*Arrays of four tiers, each tier comprising *m* elements each λ/2 long, the tiers being spaced λ/2 vertically, the lowest tier being λ above ground level.

wavelengths the radiation pattern would take the form shown in Fig. 9(b); its zeros would then be more closely spaced because of the greater separation between the array and its image. Thus if an Hm/4/1 array is erected near the edge of a plateau 3·5 wavelengths high its v.r.p. would be expected to resemble Fig. 9(b) at low angles, because reflexion takes place in the lower ground beyond the plateau. At high angles the v.r.p. should resemble Fig. 9(a). The range of angles over which the transition between the two patterns takes place depends on the distance of the array from the edge of the plateau; the shape of the pattern in this intermediate region cannot be determined by ray theory, but may be computed by the method described in this report.

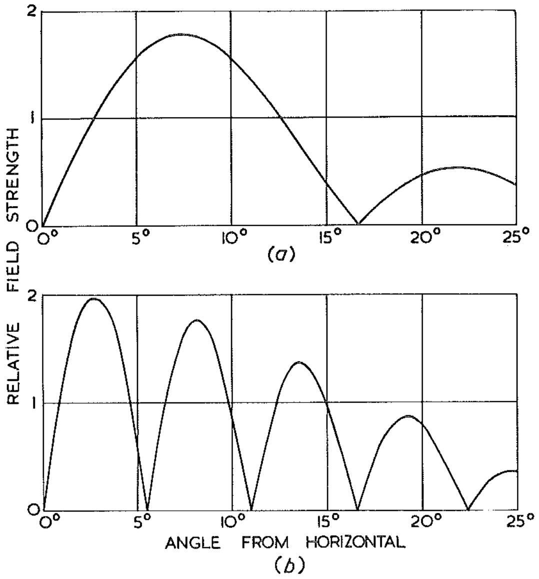

Fig. 9 - Theoretical vertical radiation patterns of arrays on flat sites

(a) an Hm/4/1 array

(b) the same array raised an additional 3·5λ above ground

Radiation patterns were computed for an array at a number of different distances from the edge of a plateau having a slope of 6·7°, since this corresponds approximately to the gradient at the east of the Daventry site. The results of these computations are shown in Fig. 10. Each pattern is compared with the theoretical v.r.p. for a flat site, i.e. Fig. 9(a), and all the patterns are drawn to the same scale,* the currents in the radiating elements being assumed to remain constant. The patterns therefore give an approximate indication of the relative field strengths established for equal radiated powers, because changes of ground slope will not greatly affect the impedance of the array.

It will be seen that when the array is close to the edge of the plateau the pattern bears less resemblance to Fig. 9(b) than might have been expected. This discrepancy is due to reflexion in the sloping ground beyond the edge; this enhances the lowest maximum and reduces the other maxima. In order to determine how this effect varies with the steepness of the slope, further computations were performed for plateaux of differing gradient. The results of these computations are shown in Fig. 11 and it will be seen that reflexion in the lower ground plane is gradually suppressed as the sloping plane extends further out. Eventually the pattern resembles Fig. 9(a), but with a slight reduction in the angle of maximum radiation, consistent with the gradient of the sloping plane.

The radiation patterns of Figs. 10 and 11 enable a few rules concerning the siting of arrays near the edges of plateaux to be formulated. At first sight it would appear to be an advantage to place an array near the edge of a steep plateau because, as Fig. 10 shows, the maximum field strength radiated by the array is considerably increased. However, this enhanced field strength occurs at too low an angle, and over too narrow a range of angles, to be advantageous. It has been shown[2] that there is an optimum value for the maximum radiation angle which depends on the distance to the service area, and that inferior cover results if an array

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*The vertical scales of Figs. 9, 10 and 11 are such that, if the array were in free space and its current were unchanged, it would radiate unit field in the direction normal to it.

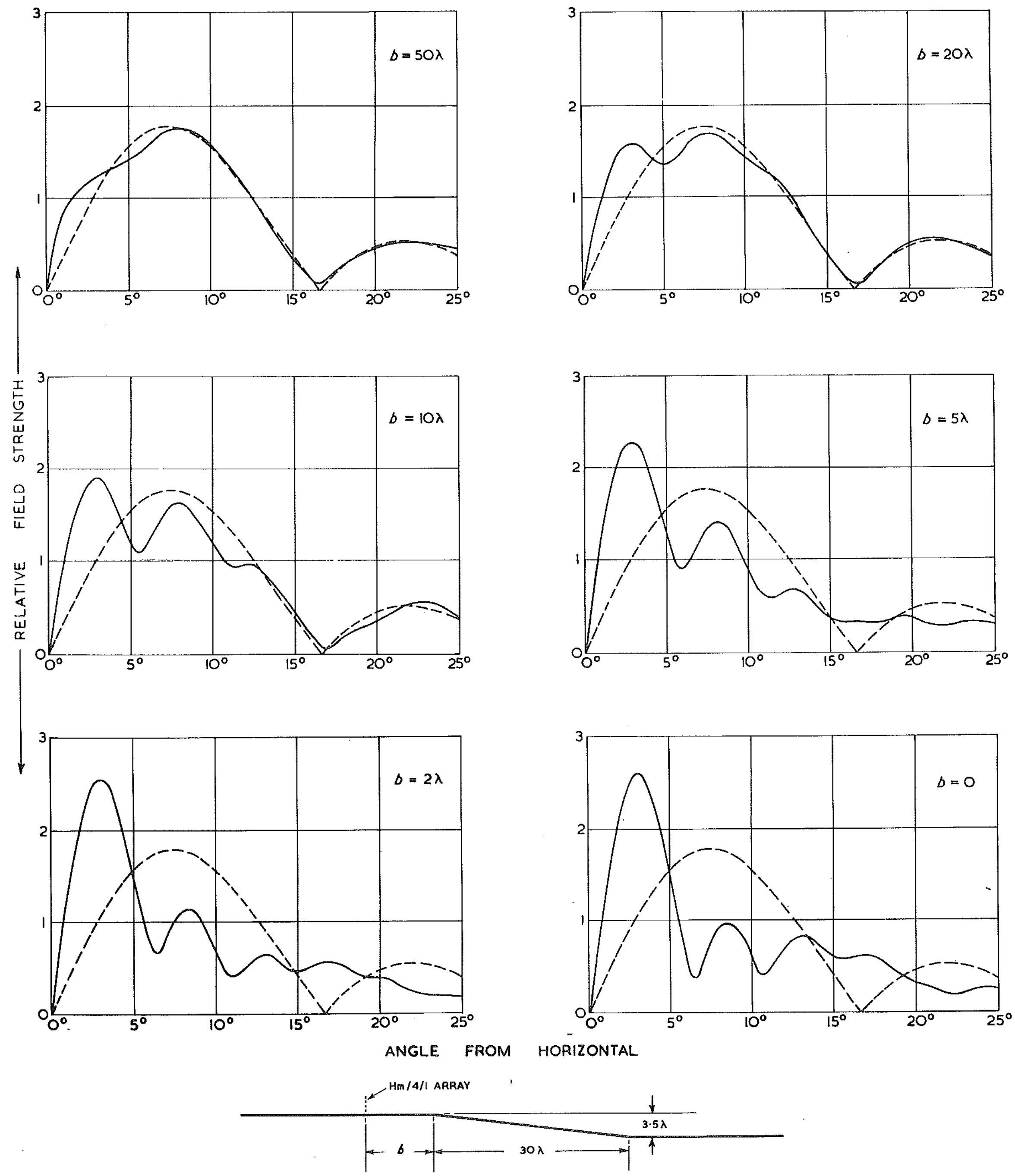

Fig. 10 - Computed vertical radiation patterns of an Hm/4/1 array at different distances from the edge of a plateau

—— computed v.r.p.

– – – theoretical v.r.p.
for a flat site

with a lower radiation angle is used, even though it may have a higher gain. We will therefore assume that the most suitable type of array for the distance to be served has been selected.[2] It should then be sited in such a way that its v.r.p. does not differ appreciably from the flat earth pattern.

Inspection of Fig. 10 shows that this condition is satisfied in the case of an Hm/4/1 array provided it is situated at least 20λ from the edge of a plateau of moderate gradient.* The ground reflexion point for a ray originating at the centre of the array, at an angle of 7·5° to the horizontal (the maximum radiation angle for the array on a flat site), then lies on the level ground at the top of the plateau.

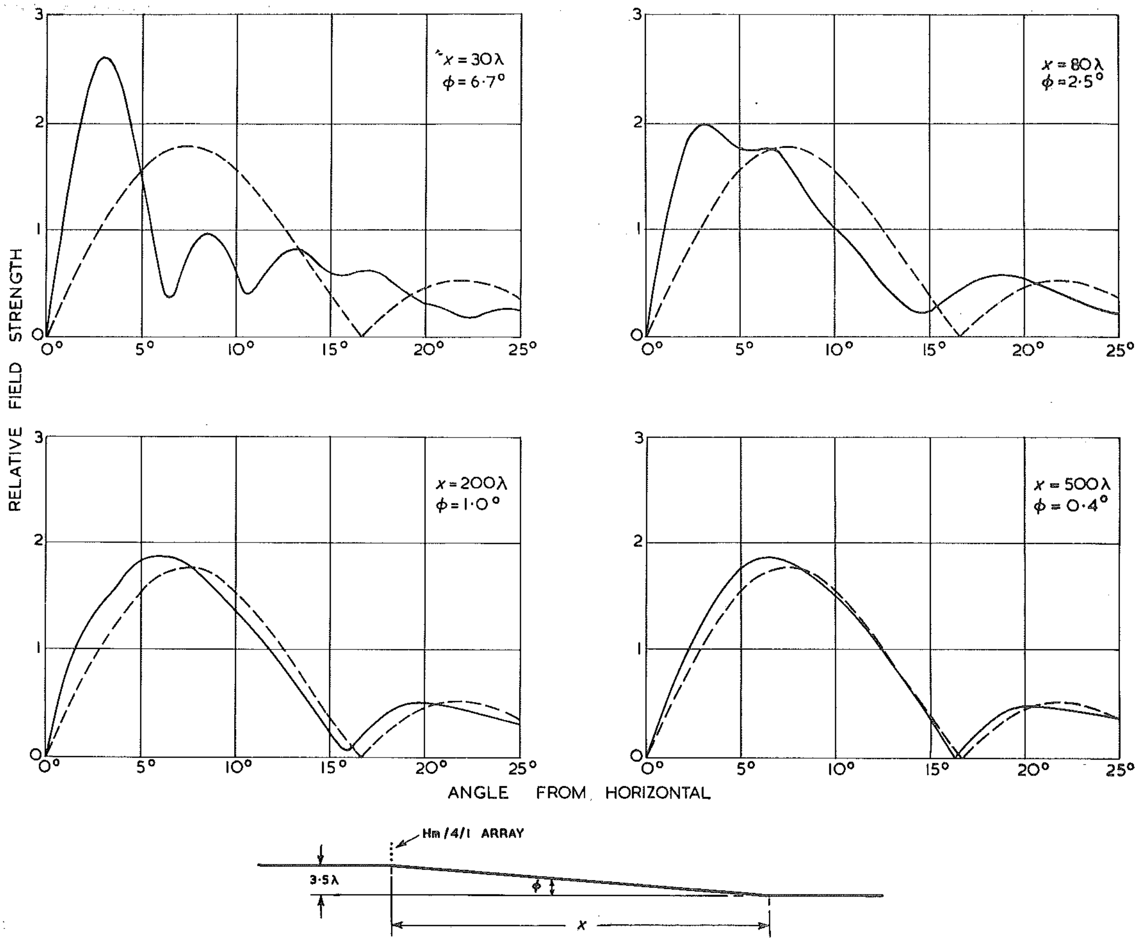

Fig. 11 - Computed vertical radiation patterns of an Hm/4/1 array at the edge of plateaux of different gradient

—— computed v.r.p.

– – – theoretical v.r.p. for a flat site

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*It is of interest to note that in Reference 1 the conclusion was reached that, for a sloping site, the angle of maximum radiation is determined by the average slope over the initial 25λ in front of the array.

This suggests that the rule which should be adopted for the siting of any array is that the ground should be level in the direction of propagation (or, if sloping, of constant gradient, which is taken into account in designing the array) for at least as far as the reflexion point for rays originating at its centre. It can be shown that this condition is satisfied provided $b/\lambda > 1/4 \sin\Delta\tan\Delta$ where b is the distance to the edge of the plateau and Δ is the angle of maximum radiation. Fig. 12 shows b/λ plotted as a function of Δ. It will be seen that the minimum distance decreases rapidly as Δ increases, because the height of the mid-point of an array designed to radiate at a given value of Δ also decreases as Δ increases.

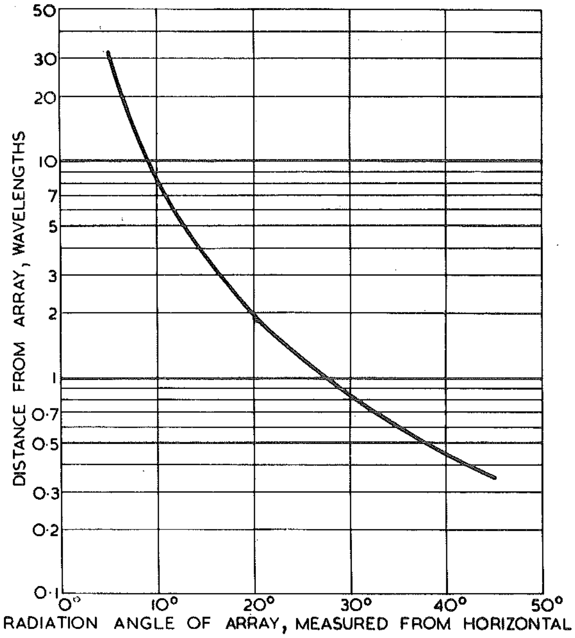

Fig. 12 - The minimum distance for which the ground should be level in front of an array

Fig. 12 should only be regarded as a provisional guide, because the distance to the edge of the plateau can sometimes be decreased. For example, Fig. 11 shows that the array can be situated right at the edge of the plateau when the slope of the latter is small compared with the maximum radiation angle. An array could also be placed close to the edge of a very low plateau; in this case the effect of the plateau might be partly compensated by reducing the height of the mid-point of the array above the ground.

5. CONCLUSIONS

It has been shown that the method of computation described in this report provides an accurate method for calculating the v.r.p. s of h.f. curtain arrays on typical plateau sites. From the computations that have been performed it has been concluded that the v.r.p. of an array will not differ appreciably from its 'flat site' v.r.p. if level ground extends in the direction of propagation for not less than the distance shown in Fig. 12. This should not, however, be regarded as a hard-and-fast rule because in some circumstances it can be relaxed. It is therefore recommended that the radiation pattern of any array which it is proposed to erect on a plateau site should be individually computed, using the parameters which most closely define the ground profile. This will give a more reliable guide to its performance than a set of rules derived from computations of a large number of hypothetical cases.

6. REFERENCES

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